Wavelet-based Shape-Feature Extraction for GPR Detection of Non-Metallic Anti-personnel Land Mines

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ABSTRACT

This paper describes a novel approach to detecting shape features of non-metallic anti-personnel land mine (NM APL) using certain attributes of wavelet packets and wavelets in general. Data sources included 2GHz and 10 GHz pulse ground penetrating radar (GPR) data, and 6 GHz stepped-frequency GPR data, from laboratory measurements. Targets are chosen to include rocks and other non-lethal clutter which normally present false alarms to the GPR. The GPR signals are first de-noised with an adaptive wavelet packet de-noising method, then the shape features of the APL are determined from exploiting the regularity properties of wavelets. Results are shown which indicate that the method may be applied to pulse GPR processing, but not for frequency-stepped GPRs. Preprocessing the GPR signal for clutter reduction may not always be necessary, thus simplifying the detection process.

Keywords: GPR, anti-personnel land mine, wavelet, singularity, demining, entropy, best basis

1. INTRODUCTION

Anti-personnel landmines (APLs) generally have flat top or bottom surfaces which may provide exploitable features to a ground penetrating radar (GPR). In addition, most have some internal structure, including air spaces, which may provide some identifiable features. On the other hand, non-lethal clutter such as stones, shrapnel, etc., have neither regular shape features nor internal structure. This paper provides a novel approach to highlighting the presence of flat surfaces of potentially lethal targets while simultaneously reducing the signature of non-lethal clutter targets.

It is often recommended that a GPR be integrated with a metal detector to resolve false alarms; this may not always be effective. The author has personally observed that, in many practical demining environments, stones and other apparently non-metallic targets present a false alarm to metal detectors. This was observed in Cambodia, for example, where stones at an abandoned campfire site caused the metal detectors to alarm. Thus it is important that the GPR have a built-in capability to address such an environment.

1.1. The Environment

APLs remain hidden in the ground in over 64 countries following the termination of armed conflict.[1–3] The exact number of APLs is not known[4]; indeed, the actual number of APLs is rather irrelevant. The fact remains that APLs account for hundreds of civilian casualties per year and prevent the return of land to agricultural use The standard approach to the detection of APLs remains the metal detector (MD) which is essentially unchanged from the approach used in World War II. Because a large number of APLs contain little to no metal, ground penetrating radar (GPR) is one of the current technologies which is receiving attention as an alternative or adjunct to the metal detector. The requirement for reliable landmine detection from a stand-off location is applicable for both military and humanitarian (post-conflict) applications. Comprehensive reports on current research in APL detection/classification are found in the proceedings of the First and Second IEE/EUREL International Conferences on the Detection of Abandoned Land Mines in Edinburgh, Scotland, in October 1996 and October 1998,[5,6] and the proceedings of the October 1997 conference on Sustainable Humanitarian Demining (SusDem ’97) held in Zagreb, Croatia.[7] SPIE, the International Society for Optical Engineering has held four conferences on demining technology[8–11]; over 250 papers are contained in those volumes covering all aspects of mine detection technologies.

For humanitarian demining, the GPR is severely constrained to be man-portable; hence, a number of system trade-offs must be made. Large phased arrays are out of the question, and vehicle-mounted systems are most likely incompatible with terrain in post-conflict developing countries.[12] While the indigenous operator is usually well-trained, he may be limited in his ability to operate high-tech equipment; thus, any detection/classification device must present the operator with unambiguous information. Signal and image processing algorithms used for such demining must aid the operator in achieving very high

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levels of detection, currently 99.6 to 99.9%.[13,14] False alarm rates must also be reduced. False alarm rates in Afghanistan, for example, using standard manual demining techniques, approach 1,000:1.[15] Personal experience by this author in Cambodia confirm this statement; the vast majority of time is spent in detecting/removing shrapnel and other non-lethal debris.

The current method of demining is quite simple, effective, but very slow. Figure 1 shows the environment in many parts of the world. Once an area is mined, grass and other shrubbery rapidly covers the area and must be removed, often by hand. The figure illustrates the common method of searching for mines with a metal detector. The deminer operates within a lane approximately 1 meter wide as indicated by the strips of cloth on either side of the deminer. Figure 2 shows the author in Cambodia, demonstrating the prodding technique; the 1 meter wooden stick denotes the boundary of the cleared lane.

1.2. APL Detection with Ground Penetrating Radar

A basic reference book on GPR is provided by Daniels,[16] and includes details of several contemporary GPR devices with application to APL detection. Other general papers on the applications of GPR and basic radar phenomenology related to sub-surface investigations are also available in Refs. 17–25. A brief tutorial on modern signal processing applications to APL detection/classification is given by the author.[26] Vitebskiy and Carin[27] derive field equations for dielectric bodies buried in soil and show that the complex natural resonances of such bodies are dependent on the burial depth. Vitebskiy *et al.*[28] develop a 3-D Method of Moments simulation for buried conducting targets of revolution. This is further extended in Geng, *et al.*[29,30] in which ultra wideband radar responses are modeled for buried dielectric bodies of revolution at various orientations. Bourgeois and Smith[31] compare the results of a FDTD simulation of buried dielectric bodies with experiments. Langman *et al.*[32] derive radar backscatter fields over a dielectric halfspace and propose target-independent scattering parameters for target classification. Cloude, *et al.*[33] show the differences in arrival times of various internal structures of the dielectric target based on the various ray paths within the target. Jaureguy, *et al.*[34] attempt to apply complex natural resonance theories to the classification of mine-like objects. References 35–38 describe feature extraction methods applied to automatic target recognition. Brunzell[39] covers a number of feature extraction methods including principal component analysis and some time-frequency distributions. Sahli, *et al.*[40] and van Kempen *et al.*[41] have developed some very promising classification algorithms using the measured DeTec data.

Figure 1. Deminer in the Tall Grass, Clearing Lane

Figure 2. Author Prodding for APL in Cambodia

2. WAVELET-BASED TARGET FEATURE DETECTION

The main premise of this paper is that the mine, being a man-made object, has certain geometrical features such as flat surfaces which may be detected by observing the distributed interface between the mine and the surrounding soil. Thus, we seek to highlight the flat top and bottom surfaces of the APL.

Any irregularities in the soil will cause the GPR return to appear noisy, and it is necessary to remove some of that interference in order to improve signal detectability. The usual noise reduction method employs simple filtering with no regard given to the recovery of the original signal, except in a mean-square sense. Filtering (usually, high-pass filtering) can remove desired high-frequency components of the signal while leaving "noise" in the lower frequency bands. Wavelet packets provide a unique method of selectively removing "just enough" noise throughout the signal bandwidth. Donoho[42] provides specific criteria for evaluating the performance of a "de-noising" strategy; specifically, the smoothness of the resultant signal and the adaptivity of the thresholding process are detailed. The Best Basis"[43] method involves searching an ordered set of orthogonal (orthonormal) wavelet packet basis tree nodes and deciding which nodes may be removed while simultaneously maintaining as much information content in the signal. Such approaches are used in signal compression and de-noising, for example. As used herein, each A-scan signal is decomposed into a library of wavelet packet bases and specific thresholds based on the entropy of the signal will be applied to decide which coefficients will be removed.

Figure 2 illustrates the steps used to process the GPR data. Clutter removal may be required, and methods to accomplish this adaptively are proposed in Refs. 44–46

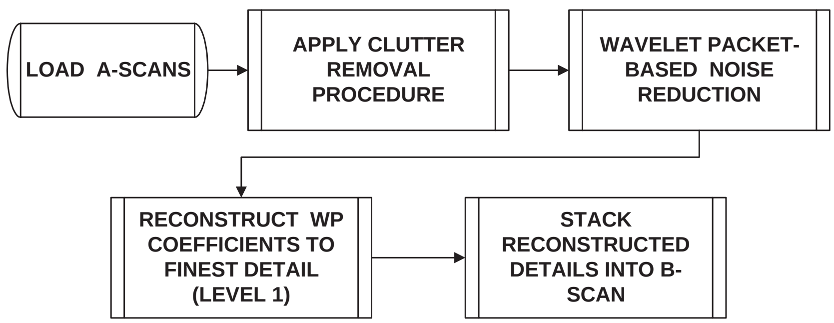

Figure 3. Procedural Flow

2.1. Wavelet Packet-Based De-Noising

It is shown in Ref.43 that the wavelet packet tree forms a *library of wavelet packet bases*; furthermore, the orthonormal wavelet packet decomposition forms a *binary tree*. In this paper, the *Best Basis* algorithm presented in Ref. 43 is used here to "prune" the wavelet packet tree. Following the construction of the Best Basis tree, an appropriate threshold is applied to the noisy coefficients. The selection of an appropriate threshold is suggested by Donoho[42] in consideration of the following property[47]: Let $\{z\}$ be an $i.i.d.$ random sequence over $N(0,1)$, then

$$Pr\left[\|\boldsymbol{z}\|_{\infty} \leq \sqrt{2\log n}\right] \rightarrow 1, \qquad n \rightarrow \infty \tag{1}$$

where n is the length of the vector $\boldsymbol{z}$. Thus, a simple threshold of

$$\sqrt{2\log n}$$

may be applied to the basis tree nodes following Best Basis selection. Full details of the above procedure and algorithms are presented in Ref. 45.

2.2. Singularity Detection Using Wavelets

Wavelets which have special properties are uniquely qualified to detect singularities in functions. The fundamental property of the wavelet $\psi(t)$ was treated by Jafard[48] who established the following theorem which requires only that ψt have n vanishing moments, is n-times continuously differentiable and has compact support.

Theorem 1 (Regularity at a Point). *Let $n \in \mathbb{Z}_+$, $\alpha < n$. Let $f(t) \in L^2(\mathbb{R})$. Let*

$$\left(\mathcal{W}_\psi f\right)(b,a) = |a|^{-1/2} \int_{-\infty}^{+\infty} f(t)\psi\left(\frac{t-b}{a}\right) dt = \langle f, \psi \rangle \tag{2}$$

be the wavelet transform of $f(t)$. If $f(t)$ is Lipschitz α at t_0, then there exists a constant A such that for all points t in the neighborhood of t_0 and any scale j

$$\left| \left(\mathcal{W}_\psi f\right)(t,j) \right| \leq A(j^\alpha + |t-t_0|^\alpha). \tag{3}$$

Let $\alpha < n$ be a noninteger. Then the function $f(t)$ is Lipschitz α at t_0, if the following two conditions apply:

(i) There exists some $\epsilon > 0$ and a constant A such that for all points t in a neighborhood of t_0 and any scale j

$$\left| \left(\mathcal{W}_\psi f\right)(t,j) \right| \leq As^\epsilon. \tag{4}$$

(ii) There exists a constant B such that for all points t in a neighborhood of t_0 and any scale j

$$\left| \left(\mathcal{W}_\psi f\right)(t,j) \right| \leq B\left(j^\alpha + \frac{|t-t_0|^\alpha}{|\log|t-t_0||}\right). \tag{5}$$

Theorem 1 proves that the wavelet transform is well-suited to provide a measure of singularities of a function. In this paper, those singularities correspond to the interfaces between the mine and the surrounding medium. In practice, the wavelet packet decomposition from the de-noising step is reconstructed to the first scale, and stacked into a B-Scan or C-Scan for further processing.

3. RESULTS

The following summarize the results of the wavelet-based de-noising and edge detection techniques described above. Clutter background was adaptively removed in accordance with the method developed in Ref. 44. It is shown that details of the mine targets are emphasized while simultaneously reducing the shape features of false targets. This is also shown to be a function of the GPR pulse bandwidth, with improved results at the wider instantaneous bandwidth of 10 GHz.

3.1. 2 GHz Pulse Data From VUB Laboratory

The experiments conducted by VUB personnel and the author in the Spring of 1999 were explicitly designed to include numerous additional targets in the soil and sand in order to achieve a more reliable method of comparing processing approaches. The experience in Cambodia served a very useful purpose in that it made clear the need to collect data which includes several target types. Figures 4 and 5 show the ability of the wavelet-based processing to suppress the false target while highlighting the features of the mine.

3.2. 10 GHz Pulse Data From RMA

The 10GHz data from RMA reflects the benefit of a very wide bandwidth pulse, but the data is limited in that there are only either mines or rocks in the data sets, none appear simultaneously. So, only relative performance of the Best Basis approach can be assessed at this time, but the results appear very promising. Figures 6 and 7 show a slice of a 3-D image of the mine and stone created by visualizing the 3-D scanned volume. The stone is simply missing; residual surface clutter appears in both figures.

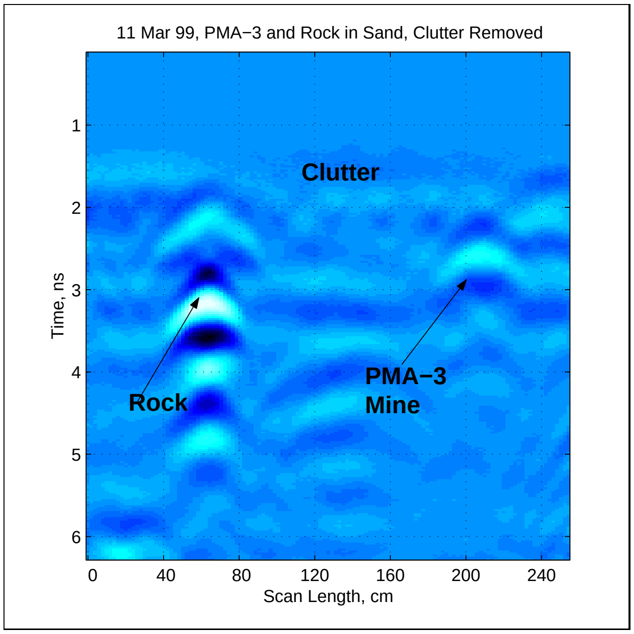

Figure 4. Rock and Mine

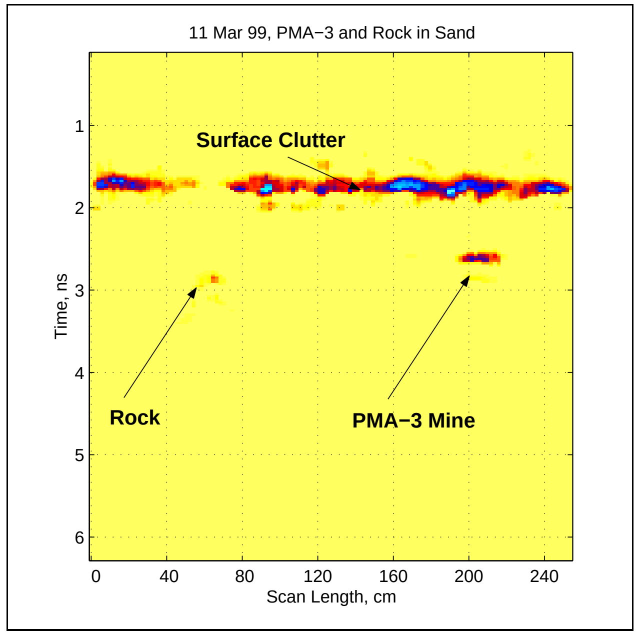

Figure 5. Wavelet Approach, Suppression of False Target

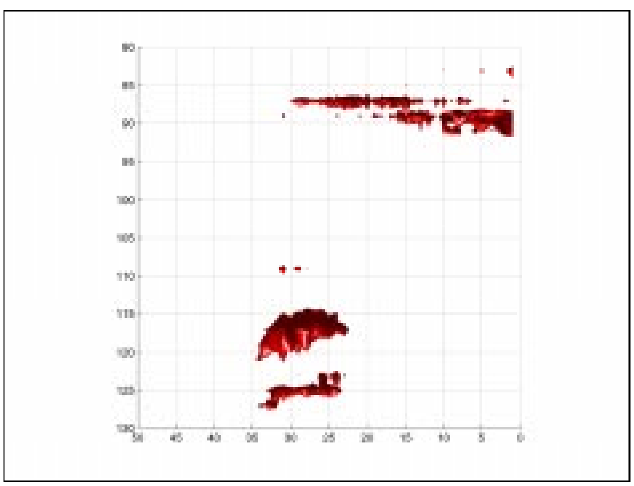

Figure 6. PMN Mine Volume, Side View

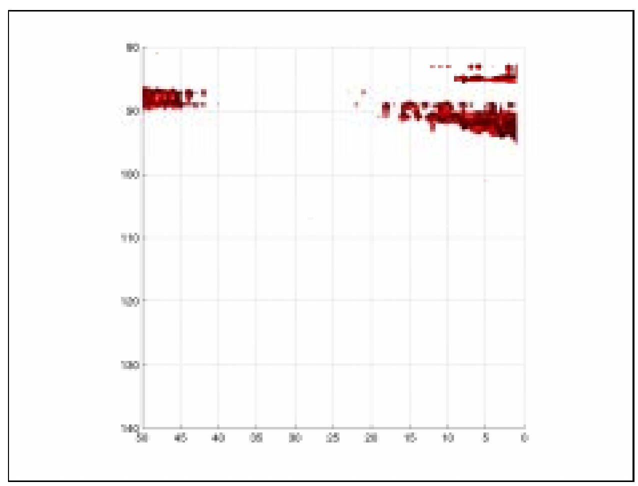

Figure 7. Stone Volume, Side View

3.3. 6 GHz Data from TUI

The results from applying the Best Basis approach to the frequency-stepped GPR data collected at TUI were inconclusive. Figure 8 shows the results of applying the Best basis approach to PMA-3 and PMA-1 APLs and a stone and aluminum sphere. Possible causes for this behavior are being further investigated.

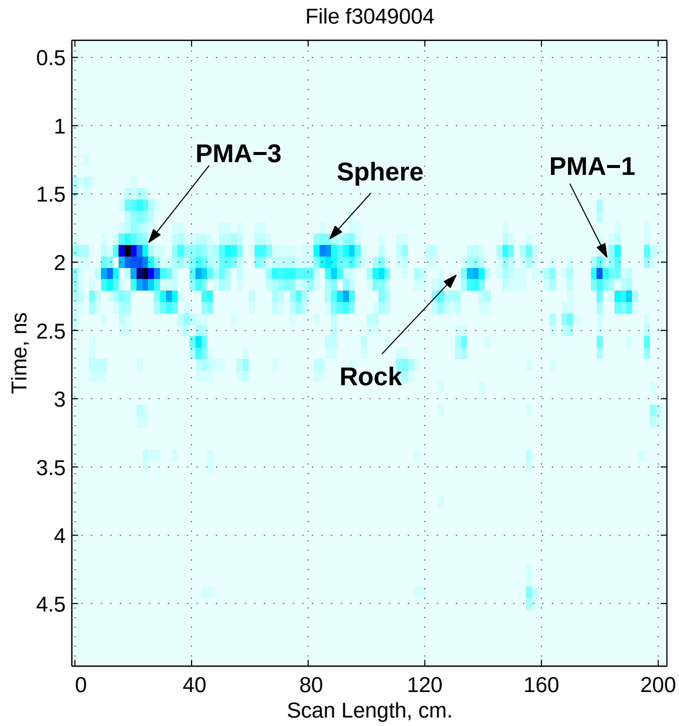

Figure 8. Best Basis Results With TUI Frequency-Stepped GPR

4. CONCLUSIONS

This paper describes a novel method of exploiting the physical features of nonmetallic and minimum-metal APLs. The method involves first the reduction of noise in individual A-Scans by the Best-Entropy Basis approach to wavelet packets; this is then followed by exploiting the properties of the wavelet transform to detect the edges and internal structure of the APL. Example of false target suppression were provided. Very promising results have been achieved particularly with ultra wideband pulse GPR.

APPENDIX A. DATA SOURCES AND COLLECTION

The GPR data files used in this paper were obtained from a number of sources, including the laboratories of the Vrije Universiteit Brussel (VUB), Technische Universität Ilmenau (TUI) and the Belgian Royal Military Academy (RMA).

A.1. DeTec Hardware

The hardware used at the VUB `http://etros1.vub.ac.be/minedet/` was based on the SPRScan commercial system, by ERA Technology Ltd.(UK). A portable version of this system was also used in Cambodia in November 1997. The SPRScan acquires a maximum of 195 A-scans, of 512 points each, with 16 bit resolution and a maximum equivalent sampling rate of 40 GHz (25 ps time resolution). A prototype resistively loaded parallel dipole antenna was used for the data acquisition (size: 195 x 195 x 95 mm). The pulse generator (pulse width: 200 ps, repetition rate: 1 MHz) is integrated into the antenna case to minimize losses and transmission reflections. This antenna has a nominal bandwidth of 800 MHz to 2.5 GHz, which leads to an expected resolution of less than 5 cm.

In March and April 1999, the author participated in extensive GPR data collection at the VUB with his colleague, Luc van Kempen, under the direction of the VUB Program Manager, Dr. Hichem Sahli. This data collection effort was in support of the EU humanitarian demining program DEMINE. The equipment used was the identical DeTec laboratory equipment described above. The objectives of this series of measurements was to extend the target sets to take advantage of the lessons learned from the Cambodia collections; that is, include numerous interfering targets such as roots, rocks and additional material which all can cause serious false alarms with the GPR. In Cambodia, it was found that a realistic minefield environment includes numerous targets which confuse both metal detectors and GPRs. For example, the metal detector often alarmed when common firebricks were encountered. In other cases, moist roots and rocks appeared to the GPR to be probable mines. Therefore, in the VUB tests, actual metal clutter and a rock taken from Cambodia were used as interfering targets.

The VUB laboratory setup included, in addition to an area containing sand, a test area consisting only of a clayey soil. This soil also had an uneven surface to more closely approximate real-world conditions. Finally, to provide additional realism, the soil was soaked with water and GPR measurements were made of the test area immediately after the soaking and also after three weeks had passed to permit some drying of the soil surface. All data files are available at the VUB website.

A.2. Data Collected at the Belgian Royal Military Academy (RMA)

The author was provided wideband (10 GHz) pulse GPR data which had been collected at the Belgian Royal Military Academy in the Summer of 1999. Dr. Marc Acheroy and Major Bart Scheers kindly provided the data which was collected using an experimental antenna designed at the RMA.

A.3. Data Collected at The Technische Universität Ilmenau (TUI)

An objective of the test was to demonstrate the viability of a stepped frequency GPR in contrast to the more standard pulse GPR. Prof. Jürgen Sachs of TUI directed the overall collection program under the auspices of the EU DEMINE Program. The sandbox is approximately 2.2 m long and 0.75 m wide. This permits up to four separate targets to be placed in such a manner to ensure at least 50 cm spacing between the targets. The data was collected by dwelling at a particular position in a grid and sweeping from 0 to 6 GHz in 15 MHz steps, for a total of 401 frequency samples per A-Scan.

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